RAN-2203000206020031

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T.Y.B.Sc. (Semester-VI) Examination March - 2026

Mathematics Paper : XVII-MTH-601-Old Course - Ring Theory

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book.

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

Seat No.:

Student's Signature

Q-1. Answer the following as directed: (Any FIVE) **10**

1) Define Field. Give an example of an integral domain; which is not a field.
2) In a Boolean ring R; prove that $a + a = 0$; for every a in R.
3) Define Left Ideal and Right Ideal in Ring.
4) If U is an ideal of a ring R with a *unit* element 1 and $1 \in U$ then prove that $U = R$.
5) Define and illustrate: the divisibility relation $a \mid b$ in a commutative ring.
6) Define *Associates* in a Commutative Ring with a *Unit* Element. Which is an *associate* of 0; the identity for addition in a commutative ring R?
7) Define Prime Element and Relatively Prime Elements in a Euclidean Ring.
8) For elements a, b in a Euclidean ring R; if π is a prime element in R and then prove that $\pi \mid ab$ then prove that $\pi \mid a$ or $a \mid b$.

Q-2. Answer any Two of the following: **10**

1) Define Integral Domain. Prove that every field is an integral domain.
2) Prove that the commutative ring D is an integral domain if and only if $a, b, c \in D$ with $a \neq 0$; the relation $a \cdot b = a \cdot c \Rightarrow b = c$ holds in D.
3) Give its one illustration of a Boolean ring. Prove that every Boolean ring is commutative.

Q-3. Answer any Two of the following: **10**

1) Define Homomorphism $\phi : R \to R'$ of a ring R into a ring R' Let $\phi : R \to R'$ be a homomorphism of a ring R into a ring R' . Then prove that :–
   i) $\phi(0) = 0'$;
   ii) $\phi(-a) = -\phi(a)$; for every a in R.
2) Define Ideal in Ring. If F is a field, then prove that its only ideals are (0) and F itself.

3) Let a be a fixed element of a commutative ring R. Then prove that $r(a) = \{x \in R \mid a \cdot x = 0\}$ is an ideal of R.

Q-4. Answer any Two of the following: **10**

1) Give an example of a Euclidean Ring; which is not a field. Prove that every field is a Euclidean ring.
2) If, for a, b in an integral domain R with a *unit* element; both $a \mid b$ and $b \mid a$ hold true, then prove that a, b are *associates* in R.
3) Let R be a Euclidean ring and $a \neq 0$, $b \neq 0$ in R. If b is not *unit* in R, then prove that $d(a) < d(ab)$.

Q-5. Answer any Two of the following: **10**

1) If a, b and c are elements in a Euclidean ring R such that $a \mid bc$; where a, b relatively prime elements, then prove that $a \mid c$.
2) For elements a, b in a Euclidean Ring R; prove that :–
   i) If $(a, b) = d_1$ and $(a, b) = d_2$, then d_1 and d_2 are *associates*.
   ii) If $b \neq 0$ is *unit* in R, then $d(a) = d(ab)$.
3) Prove that the relation of "*associates*" in a commutative ring R with a *unit* element is an equivalence relation on R.